Mean
Variance
Standard Deviation

Binomial
Probability Distribution

What is **Binomial Probability Distribution**?

It is a probability distribution for a discrete random variable x with probability $P(x)$ such that

- $\sum P(x) = 1$.
- $0 \leq P(x) \leq 1$.
- It has a fixed number of independent events.
- Each event has only two outcomes, and are referred to as success and failure.
- The probability of success and failure remains the same for all events.

Binomial Probability Distribution Notations:

- Number of independent trials $\Rightarrow n$
- Number of successes $\Rightarrow x$
- Probability of success in one of the trials $\Rightarrow p$
- Probability of failure in one of the trials $\Rightarrow q$ where $q = 1 - p$

Finding **Mean**, **Variance**, and **Standard Deviation**

of **Binomial Probability Distribution**

Given a binomial probability distribution for n independent events with probability of success p, then

- Mean $\Rightarrow \mu = n \cdot p$,
- Variance $\Rightarrow \sigma^2 = n \cdot p \cdot q$, and
- Standard Deviation $\Rightarrow \sigma = \sqrt{\sigma^2}$.

Example:

Given: Binomial probability distribution with $n = 30$, and $p = .6$. Find its mean, variance, and standard deviation, then its usual range.

Solution:

We first find
$q = 1 - p = 1 - 0.6 = 0.4$,
Now we can use formulas introduced earlier to find μ, σ^2, and σ.

$$\begin{aligned} \mu &= n \cdot p \\ &= 30 \cdot 0.6 \\ &= 18 \end{aligned}$$

Solution Continued:

$$\begin{aligned}\sigma^2 &= n \cdot p \cdot q \\ &= 30 \cdot 0.6 \cdot 0.4 \\ &= 7.2\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\sigma^2} \\ &= \sqrt{7.2} \\ &\approx 2.683\end{aligned}$$

$$\begin{aligned}\mu \pm 2 \cdot \sigma &\Rightarrow 18 \pm 2 \cdot 2.683 \\ &\Rightarrow 18 \pm 5.366 \\ &\Rightarrow 12.634 \text{ to } 23.366 \ \underline{\text{Usual Range or 95\% Range}}\end{aligned}$$

Example:

Given: Binomial probability distribution with $n = 100$, and $p = .5$. Find its mean, variance, and standard deviation, then its 68% range.

Solution:

We first find
$q = 1 - p = 1 - 0.5 = 0.5$,
Now we can use formulas introduced earlier to find $\mu, \sigma^2,$ and σ.

$$\begin{aligned} \mu &= n \cdot p \\ &= 100 \cdot 0.5 \\ &= 50 \end{aligned}$$

Solution Continued:

$$\begin{aligned} \sigma^2 &= n \cdot p \cdot q \\ &= 100 \cdot 0.5 \cdot 0.5 \\ &= 25 \end{aligned}$$

$$\begin{aligned} \sigma &= \sqrt{\sigma^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \mu \pm \sigma &\Rightarrow 50 \pm 5 \\ &\Rightarrow 45 \text{ to } 55 \ \underline{68\% \text{ Range}} \end{aligned}$$

Example:

An online real estate school claims that 80% of the students who have completed their courses would pass the state sponsored exam. If we randomly select 400 of their students that have completed successfully their training and course contents,

1. what is the mean for number of students that they successfully pass the state sponsored exam?
2. what is the standard deviation for the number of students that pass the state sponsored exam?
3. what is the usual range for the number of students that they successfully pass the state sponsored exam?

Solution:

We first obtain from the problem that $n = 400$, $p = 0.8$, and $q = 0.2$. Now

Solution Continued:

1. what is the mean for number of students that they successfully pass the state sponsored exam?

$$\begin{aligned} \mu &= n \cdot p = 400 \cdot 0.8 \\ &= 320 \end{aligned}$$

2. what is the standard deviation for the number of students that pass the state sponsored exam?

$$\begin{aligned} \sigma &= \sqrt{\sigma^2} = \sqrt{n \cdot p \cdot q} = \sqrt{400 \cdot 0.8 \cdot 0.2} = \sqrt{64} \\ &= 8 \end{aligned}$$

3. what is the usual range for the number of students that they successfully pass the state sponsored exam?

$$\begin{aligned} \mu \pm 2 \cdot \sigma &\Rightarrow 320 \pm 2 \cdot 8 \\ &\Rightarrow 320 \pm 16 \\ &\Rightarrow 304 \text{ to } 336 \ \underline{\text{Usual Range or 95\% Range}} \end{aligned}$$

Example:

An airline claims that 95% of passengers with tickets actually show up for the flight. The airline overbooked a flight by selling 140 tickets for the aircraft that has 136 seats. Find the mean and standard deviation for the number of passengers that have purchased tickets that actually show up the flight.

Solution:

We first obtain from the problem that $n = 140$, $p = 0.95$, and $q = 0.05$. Now we can use formulas to find μ, and σ.

$$
\begin{aligned}
\mu &= n \cdot p = 140 \cdot 0.95 \\
&= 133 \\
\sigma &= \sqrt{\sigma^2} = \sqrt{n \cdot p \cdot q} = \sqrt{140 \cdot 0.95 \cdot 0.05} = \sqrt{6.65} \\
&\approx 2.579
\end{aligned}
$$